February 7, 2008

Rotation in classical zero-point radiation and in quantum vacuum.

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Abstract

Two reference systems, rotating $\{\mu_\tau\}$ and non rotating $\{\lambda_\tau\}$, are defined and used as the basis for investigating thermal effects of rotation through both random *classical* zero point radiation and *quantum* vacuum. Both reference systems consist of an infinite number of inertial reference frames μ_τ and λ_τ respectively. The μ and λ reference frames do not accompany the detector and are defined so that at each moment of proper time τ of the detector there are two inertial frames, μ_τ and λ_τ, which agree momentarily, are connected by a Lorentz transformation with the detector velocity as a parameter, and with origins at the detector location at the same time τ.

The two- field correlation functions measured by the observer rotating through a random classical zero point radiation , have been calculated and presented in terms of elementary functions for both *electromagnetic* and *massless scalar* fields.

If the correlation functions are periodic with a period $\frac{2\pi}{\Omega}$ of rotation the observer finds the spectrum which is very similar, but not identical, to Plank spectrum.

If both fields of such a two-field periodic correlation function, for both electromagnetic and massless scalar case, are taken at the same point then its convergent (regularized) part is shown, using Abel-Plana summation formula, to have Planck spectrum with the temperature $T_{rot} = \frac{\hbar\Omega}{2\pi k}$.

The convergent (regularized) part of the electromagnetic energy density at the rotating detector is shown to have Plank spectrum $reg\ w(\mu) = \frac{2(4\gamma^2-1)}{3}\ w(T_{rot})$ where $w(T_{rot}) = \frac{4\sigma}{c}T_{tot}^4$ is the energy density of the black radiation at the temperature T_{rot} and the factor $\frac{2(4\gamma^2-1)}{3}$ is a relativistic anisotropy factor.

It is shown that the vacuum of the quantized massless scalar field in rotating reference system $\{\mu_\tau\}$ is not equivalent to the vacuum of the field in the laboratory system because the respective Bogolubov transformation is not a zero.

1 Introduction

. The main issue related with a description of classical and quantum effects connected with rotation is a definition of a reference system. We will use the concepts of laboratory coordinate system, reference frames, and reference systems.

The origin of the laboratory coordinate system is chosen at the center of the rotating detector circle. At each proper time of the rotating detector an inertial reference frame μ_τ exists. The reference frame μ_τ is a global 3-dimensional orthogonal system with constant vector velocity $\vec{v}_\tau$ relative to the laboratory coordinate system. The index τ means that μ_τ is used by the detector only once, momentarily, at the time τ and the origin of μ_τ is instantaneously at rest relative to the rotating detector and coincides with it at the time $t_\tau = \tau$, where $\vec{v}_\tau$ is the vector velocity of the detector at the proper time τ, and t_τ is the time measured in the reference frame. The first axis of the reference frame is directed along an instantaneous radius of the detector r_τ in the laboratory coordinate system. The second one is directed along the instantaneous velocity vector $\vec{v}_\tau$ of the detector in the laboratory system and the third one is perpendicular to the plane of the detector rotation. Such reference frames can be defined at each proper time τ. The description of zero-point vacuum fluctuations in the global reference frame is restricted in our consideration to a Minkovski space-time with pseudo-Cartesian coordinates.

A rotating reference system $\{\mu_\tau\}$is defined as a set of reference frames μ_τ for all possible values of τ.

A given RF, μ_τ, is connected by a Lorentz transformation, with velocity $|\vec{v}_\tau| = \Omega r$ depending on radius of rotation , to another RF , λ_τ. The index τ means again that this RF $\lambda\tau$, its origin and axis directions , agrees instantaneously with μ_τ at detector proper time τ. Each RF λ_τ is a global one, at rest relative to the laboratory coordinate system, and related with the laboratory system by spatial shift and rotation transformations. A set of all λ_τ defines a non rotating reference system $\{\lambda_\tau\}$. Both systems are used to calculate the correlation functions (CF) of the random classical electromagnetic and massless scalar fields at the rotating detector and Bogolubov transformations for a quantized massless scalar field between $\{\mu_\tau\}$ and the laboratory system.

The correlation functions (CF) of random classical fields measured by a uniformly accelerated detector have been investigated and used in many works [1, 2, 4, 5]. Our approach in CF calculation

is very close but not identical to the method developed in [1, 2] for uniformly accelerated detectors . The reference frames μ_τ are similar to the frames I_τ defined in [2], p.1091, but the reference frames λ_τ are not used by Boyer [2]. The inertial frame I_* agrees with the frame I_τ (*only*) at $\tau = 0$ [2], p. 1091. We discuss this issue in the Appendix B in detail.

The case of the rotating detector, to the best of our knowledge, has not been considered in random *classical* electromagnetic radiation. In *quantum* case it was studied for the massless scalar field in connection with rotating vacuum puzzle [9, 23, 24], Bogolubov transformation, and different coordinate mappings. In this article no special mapping between (t, r, θ, ϕ) and (t', r', θ', ϕ') is used. The Bogolubov transformation between the *quantized* massless scalar fields defined in two different reference systems is calculated based on our definitions of $\{\mu_\tau\}$ and $\{\lambda_\tau\}$. It is done in Section 4.2. The same reference systems are used for the calculation of CF of a *classical* zero-point electromagnetic zero-point radiation and a *classical* massless scalar field.

Reference system $\{\mu_\tau\}$ differs from the system of successive rest systems in constant circular motion introduced by Moller [17], IV, $47 because they have different initial conditions. This issue is also discussed in the Appendix B.

The calculation of the CF for an electromagnetic field is presented in the Section 2. The calculation of the CF for a massless scalar field is performed in the Section 4 in a similar manner, and it is much simpler than the calculation in the electromagnetic field case because the scalar field does not change under Lorentz transformations. This CF calculated in *classical* approach is identical with the correlation function of the rotating vacuum of massless scalar field, obtained in the *quantum* case [9].

There is a simple relationship between proper time of the rotating detector and the time measured in the lab system. Because of that a period $T_\gamma = T/\gamma$ can be introduced, where T_γ is the time measured by the rotating detector and which corresponds to the period of rotation, measured in the rotating reference system μ_τ. We expect that the physical picture of the vacuum observed by the rotating detector at two moments of proper time separated by T_γ is the same. For example, $CF(\tau) = CF(\tau + T_\gamma)$. This assumption is investigated below. The direct consequence of this periodicity condition is a change in the spectrum of random electromagnetic field observed by a rotating detector. It can measure the frequencies $\omega = \Omega n$ only, where $n = 0, \pm1, \pm2, \pm3, ...$ and Ω is an angular detector velocity. In the Section 3 this periodicity condition is taken into consideration, the final expression of the CF is modified, and the integration over the absolute value of the wave vector is

changed to the summation over $k_n = k_0 n = \frac{\Omega}{c} n$. Using Abel-Plana summation formula it is shown that the difference between the infinite values of the energy density with the discrete spectrum and of electromagnetic zero-point radiation with the continuous spectrum is finite at a reference frame μ and has the same spectrum as the Plank spectral function with temperature $T_{rot} = \frac{\hbar\Omega}{2\pi k}$. For the radiation, observed by the rotating detector, the simple relationship exists: $w(\mu) = w(T)\frac{4\gamma^2-1}{3\pi}$ where $w(T)$ is the Plank energy density radiation at the temperature T, excluding zero-point radiation, and the factor $\frac{4\gamma^2-1}{3\pi}$ is a consequence of the anisotropy of the radiation observed by the moving relativistic detector.

2 Electromagnetic field. Correlation functions in the case at a rotating detector.

In this section we will calculate the following two-field correlation functions measured by a detector when it experiences a rotation motion through the classical random zero-point radiation:
$\langle E_i(\tau_2)E_j(\tau_1)\rangle$, $\langle E_i(\tau_2)H_j(\tau_1)\rangle$, $\langle H_i(\tau_2)H_j(\tau_1)\rangle$,
where $i, j = 1, 2, 3$ and electric and magnetic field intensities E_i, H_i are measured by the detector at its proper times τ_1 and τ_2.

We follow the common idea [2], p.1091 that all measurements at the time τ are carried out by non inertial detector using an instantaneous inertial reference frame which is momentarily at rest relative to the detector at τ. Therefore we rewrite these expressions as follows:
$\langle E_i(\mu_2|A_2^{\mu_2}, \tau_2)E_j(\mu_1|A_1^{\mu_1}, \tau_1)\rangle$, $\langle H_i(\mu_2|A_2^{\mu_2}, \tau_2)H_j(\mu_1|A_1^{\mu_1}, \tau_1)\rangle$, and $\langle E_i(\mu_2|A_2^{\mu_2}, \tau_2)H_j(\mu_1|A_1^{\mu_1}, \tau_1)\rangle$,
So a two-field correlation function definition is based on two measurements in the instantaneous inertial reference frames μ_1 and μ_2, with the detector at the points $A_1^{\mu_1}$ and $A_2^{\mu_2}$ of μ_1 and μ_2 at proper times τ_1 and τ_2 respectively. In each reference frame μ, the time is measured by the clock of the point detector. So $\tau_1 = t_1^{\mu_1}$ and $\tau_2 = t_2^{\mu_2}$. The origin of the reference frame μ_τ is defined at the detector position at the proper time τ, so $A_1^{\mu_1} = 0$, and $A_2^{\mu_2} = 0$. But it is convenient to keep $A_1^{\mu_1}$, and $A_2^{\mu_2}$ in this unspecified form.

To execute the operation of the averaging $\langle\rangle$ of the expressions with two components of the electromagnetic field, defined in two *different* reference frames, they should be transformed to a *single* reference frame, for example λ_{τ_2} of the non rotating reference system.

The main steps of the calculation of the CF $\langle E_1(\mu_2|A_2^{\mu_2}, \tau_2)E_1(\mu_1|A_1^{\mu_1}, \tau_1)\rangle$are as follows (the same

way all other CF's can be evaluated):

1. Using Lorentz transformation, $E_1(\mu_2|A_2^{\mu_2}, \tau_2)$ defined at the point $A_2^{\mu_2}$ at the time $t^{\mu_2} = \tau_2$ in the μ_2 reference frame is expressed in terms of the electromagnetic field components, defined at the point $A_2^{\lambda_2}$ at the time $t_2^{\lambda_2}$ at the reference frame λ_2 .

2.$E_1(\mu_1|A_1^{\mu_1}, \tau_1)$, using again Lorentz transformation, is expressed in terms of the components at the reference frame λ_1 at the point $A_1^{\lambda_1}$ at the time $t_1^{\lambda_1}$. Of course the reference frames λ_1 and λ_2 have different directions and different origins in the laboratory coordinate system.

3. Using rotation transformation, all the quantities in the reference frame λ_1 are transformed to the reference frame λ_1' which, by definition, should have the same axes directions as λ_2.

4. In the next step, a shift transformation from λ_1' to λ_2 is made. After these steps, all random electromagnetic field intensities in the CF are defined in the inertial reference frame λ_2. Expressions for random field intensities in an inertial reference frame are well known and given in [1], formulae (48) and (49).

5. Using the obtained expressions, the operation $\langle\rangle$ is executed in the reference frame λ_2.

6. Summation over polarizations is made.

7. Finally the CF is expressed as 3-dimensional integral in wave vector space.

8. Its integrand expression is simplified using a rotation transformation in (k_1, k_2) plane of the wave vector space.

2.1 Expressions for field components, defined in two different reference frames, in terms of one reference frame.

We will first treat as an example with

$$\langle E_1(\mu_1|A_1^{\mu_1}, \tau_1)E_1(\mu_2|A_2^{\mu_2}, \tau_2)\rangle \tag{1}$$

and will follow the directions described above.

The fluctuating electric and magnetic fields in an inertial reference frame μ_τ or λ_τ can be written as [1]

$$\vec{E}(\vec{r}, t) = \sum_{\lambda=1}^{2} \int d^3k \hat{\epsilon}(\vec{k}, \lambda) h_0(\omega) \cos[\vec{k}\vec{r} - \omega t - \Theta(\vec{k}, \lambda)],$$
$$\vec{H}(\vec{r}, t) = \sum_{\lambda=1}^{2} \int d^3k [\hat{k}, \hat{\epsilon}^(\vec{k}, \lambda)] h_0(\omega) \cos[\vec{k}\vec{r} - \omega t - \Theta(\vec{k}, \lambda)]. \tag{2}$$

where the $\theta(\vec{k},\lambda)$ are random phases distributed uniformly on the interval $(0,2\pi)$ and independently for each wave vector $\vec{k}$ and polarization λ of of a plane wave, and

$$\pi^2 h_0^2(\omega) = (1/2)\hbar\omega. \tag{3}$$

For special Lorentz transformation without rotation between λ_i and μ_i, the transformation equations for $\vec{E}$ and $\vec{H}$ at the points A_1 and A_2 can be written in the form [17], V.(15)

$$\vec{E}(\mu_i|A_i^{\mu_i},\tau_i) = \gamma\vec{E}(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i}) + \frac{\vec{v}^{\lambda_i}}{v^2}(\vec{v}^{\lambda_i}\vec{E}(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i}))(1-\gamma) + \gamma\frac{[\vec{v}^{\lambda_i},\vec{H}(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i})]}{c}, \tag{4}$$

$$\vec{H}(\mu_i|A_i^{\mu_i},\tau_i) = \gamma\vec{H}(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i}) + \frac{\vec{v}^{\lambda_i}}{v^2}(\vec{v}^{\lambda_i}\vec{H}(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i}))(1-\gamma) - \gamma\frac{[\vec{v}^{\lambda_i},\vec{E}(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i})]}{c}, \tag{5}$$

where $i=1,2$, $\gamma=\sqrt{1-\frac{v^2}{c^2}}$ and v is a linear velocity of the detector. Its absolute value is constant, and $v=\Omega r$. Here Ω is an angular velocity of the rotating detector. Vector $\vec{v}^{\lambda_i}$ $(i=1,2$)is a velocity vector of the inertial reference frame μ_i relative to the inertial reference frame λ_i.

Because $\vec{v}^{\lambda} = (v_1^{\lambda},v_2^{\lambda},v_3^{\lambda}) = (0,v,0)$ in both reference frames, λ_1 and λ_2, we have:

$$E_1(\mu_i|A_i^{\mu_i},\tau_i) = \gamma(E_1(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i}) + \frac{v}{c}H_3(\lambda_i|A_i^{\lambda_i},t_i^{\lambda_i})),\ \ i=1,2. \tag{6}$$

For $i=1$ and $i=2$, the quantities on the right side of these equations are still defined in different reference frames, λ_1 and λ_2. Let us transform $E_1(\lambda_1|A_1^{\lambda_1},\tau_1)$ and $H_3(\lambda_1|A_1^{\lambda_1},\tau_1)$ in the last formula from λ_1 to λ_1' (which, by definition, has the same axes directions as λ_2) by rotation and then to λ_2 by shifting. Then we have

$$\begin{aligned} E_1(\mu_1|A_1^{\mu_1},\tau_1) &= \gamma(E_1(\lambda_1'|A_1^{\lambda_1'},t_1^{\lambda_1'})\cos\delta + E_2(\lambda_1'|A_1^{\lambda_1'},t_1^{\lambda_1'})(-\sin\delta) + \frac{v}{c}H_3(\lambda_1'|A_1^{\lambda_1'},t_1^{\lambda_1'})) \\ &= \gamma(E_1(\lambda_2|A_1^{\lambda_2},t_1^{\lambda_2})\cos\delta + E_2(\lambda_2|A_1^{\lambda_2},t_1^{\lambda_2})(-\sin\delta) + \frac{v}{c}H_3(\lambda_2|A_1^{\lambda_2},t_1^{\lambda_2})), \end{aligned} \tag{7}$$

where δ is an angle between λ_1 and λ_2 references frames in the laboratory coordinate system, and $\delta = \Omega\gamma(\tau_2-\tau_1)$ .The explicit expressions for the coordinates of the points A_1 and A_2 in different reference frames that is $A_1^{\mu_1}$, $A_1^{\lambda_1'}$, and $A_1^{\lambda_2}$ are provided below.

Taking into consideration (6) and (7), the CF can be written in the following form

$$\begin{aligned} \langle E_1(\mu_1|A_1^{\mu_1},\tau_1)E_1(\mu_2|A_2^{\mu_2},\tau_2)\rangle \quad &= \langle\gamma\ [E_1(\lambda_2|A_1^{\lambda_2},t_1^{\lambda_2})\cos\delta + E_2(\lambda_2|A_1^{\lambda_2},t_1^{\lambda_2})(-\sin\delta) + \\ &+\tfrac{v}{c}H_3(\lambda_2|A_1^{\lambda_2},t_1^{\lambda_2}]\ \gamma\ [E_1(\lambda_2|A_2^{\lambda_2},t_2^{\lambda_2}) + \tfrac{v}{c}H_3(\lambda_2|A_2^{\lambda_2},t_2^{\lambda_2})]\rangle. \end{aligned} \tag{8}$$

All field quantities on the right side of the equation are defined in the same reference frame λ_2 and can be substituted by the expressions (2). But first we have to specify all arguments of these quantities, that is $A_1^{\lambda_2}$, $A_2^{\lambda_2}$, $t_1^{\lambda_2}$, and $t_2^{\lambda_2}$.

2.2 Coordinates of two detector locations in terms of one reference frame.

By definition of a μ_i reference frame (we use μ_i instead of μ_{τ_i} for simplicity when such reference does not lead to confusion), its origin should be at the location of the detector and the detector proper time τ_i should be equal to $t_i^{\mu_i}$ that is

$$A_1^{\mu_1} = (x_1^{\mu_1}, x_2^{\mu_1}, x_3^{\mu_1}) = (0,0,0), \qquad t_1^{\mu_1} = \tau_1, \tag{9}$$

$$A_2^{\mu_2} = (x_1^{\mu_2}, x_2^{\mu_2}, x_3^{\mu_2}) = (0,0,0), \qquad t_2^{\mu_2} = \tau_2. \tag{10}$$

The coordinates of $\vec{A}^{\mu}$ and $\vec{A}^{\lambda}$ of the point $\vec{A}$ in two reference frames μ and λ, are connected with a Lorentz transformation without rotation and with a special initial condition (see Appendix):

$$\vec{x}^{\lambda} = \vec{x}^{\mu} + \vec{v}^{\mu}[\frac{\vec{x}^{\mu}\vec{v}^{\mu}}{v^2}(\gamma-1) - t^{\mu}\gamma] + \vec{a}_{\tau}^{\mu}, \tag{11}$$

where

$$\vec{a}_{\tau_1}^{\mu_1} = (0, v\tau_1, 0) \qquad \vec{a}_{\tau_2}^{\mu_2} = (0, v\tau_2, 0) \tag{12}$$

and v and τ_i are parameters of the transformation.

Then

$$\vec{A}_1^{\lambda_1} = (x_1^{\lambda_1}, x_2^{\lambda_1}, x_3^{\lambda_1}) = (0,0,0), \quad t_1^{\lambda_1} = \gamma\tau_1, \quad \vec{A}_2^{\lambda_2} = (x_1^{\lambda_2}, x_2^{\lambda_2}, x_3^{\lambda_2}) = (0,0,0), \quad t_2^{\lambda_2} = \gamma\tau_2. \tag{13}$$

At each moment of proper time,τ_i, the detector is at the origin of both references frames λ_i and μ_i.

After rotation from λ_1 to λ_1' and shift from λ_1' to λ_2 the coordinates of A_1 point are in λ_1' and λ_2 reference frames respectively:

$$A_1^{\lambda_1'} = (x_1^{\lambda_1'}, x_2^{\lambda_1'}, x_3^{\lambda_1'}) = (0,0,0), \;\; t_1^{\lambda_1'} = \gamma\tau_1, \;\; A_1^{\lambda_2} = (-r(1-\cos\delta), -r\sin\delta, 0), \;\; t_1^{\lambda_2} = \gamma\tau_1, \tag{14}$$

where $\delta = \Omega(t_2 - t_1) = \Omega\gamma(\tau_2 - \tau_1)$ is a rotation angle of the detector for the time $t_2 - t_1$.

The last two expressions will be used in the next subsection to get general expression to calculate (8).

2.3 General expression for the correlation function $E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle$.

Using (13) and (14) expressions (6) for $i = 2$ and (7) can be written as

$$\begin{aligned} E_1(\mu_1|A_1^{\mu_1}, \tau_1) = \gamma(&E_1(\lambda_2| - r(1-\cos\delta), -r\sin\delta, 0, \gamma\tau_1)cos\delta \\ &+E_2(\lambda_2| - r(1-\cos\delta), -r\sin\delta, 0, \gamma\tau_1)(-\sin\delta) \\ &+\frac{v}{c}H_3(\lambda_2| - r(1-\cos\delta), -r\sin\delta, 0, \gamma\tau_1)), \end{aligned} \tag{15}$$

and

$$E_1(\mu_2|A_2^{\mu_2},\tau_2)=\gamma(E_1(\lambda_2|,0,0,0,\gamma\tau_2)+\frac{v}{c}H_3(\lambda_2|0,0,0,\gamma\tau_2)). \tag{16}$$

Each field intensity component on the right sides in these expressions are defined in the single inertial reference frame λ_2. Having inserted them into (8)and using (2) we arrive at the following expression for the CF

$$\begin{aligned}\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle = \langle\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3k_1\int d^3k_2 h_0(\omega_1)h_0(\omega_2)\gamma^2\times\\ \{\hat{\epsilon}_{1x}(\vec{k}_1\lambda_1)\cos\delta+\hat{\epsilon}_{1y}(\vec{k}_1\lambda_1)(-\sin\delta)+(\hat{k}_{1x}\hat{\epsilon}_{1y}(\vec{k}_1\lambda_1)-\hat{k}_{1y}\hat{\epsilon}_{1x}(\vec{k}_1\lambda_1))\frac{v}{c}\}\times\\ \{\hat{\epsilon}_{2x}(\vec{k}_2\lambda_2)+(\hat{k}_{2x}\hat{\epsilon}_{2y}(\vec{k}_2\lambda_2)-\hat{k}_{2y}\hat{\epsilon}_{2x}(\vec{k}_2\lambda_2))\frac{v}{c}\}\times\\ \cos\{k_{1x}[-r(1-\cos\delta)]+k_{1y}(-r\sin\delta)-\omega_1\gamma\tau_1-\theta(\vec{k}_1\lambda_1)\}\cos\{-\omega_2\gamma\tau_2-\theta(\vec{k}_2\lambda_2)\}\rangle,\end{aligned} \tag{17}$$

where symbols λ_1 and λ_2 are polarizations, not reference frame labels.

Taking into consideration [1] that

$$\langle\cos\theta(\vec{k}_1\lambda_1)\cos\theta(\vec{k}_2\lambda_2)\rangle=\langle\sin\theta(\vec{k}_1\lambda_1)\sin\theta(\vec{k}_2\lambda_2)\rangle=\frac{1}{2}\delta_{\lambda_1\lambda_2}\delta^3(\vec{k}_1-\vec{k}_2) \tag{18}$$

and

$$\sum_{\lambda=1}^{2}\epsilon_i(\vec{k}\lambda)\epsilon_j(\vec{k}\lambda)=\delta_{ij}-k_ik_j/k^2 \tag{19}$$

and after integrating over $\vec{k}_1$ and summing over λ_1 this expression can be reduced to

$$\begin{aligned}\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle=\int d^3kh_0^2(\omega)\gamma^2\frac{1}{2}\times\\ (\cos\delta-\hat{k}_x\frac{v}{c}\sin\delta-\hat{k}_y 2\frac{v}{c}\cos^2\frac{\delta}{2}+\hat{k}_x\hat{k}_y\sin\delta+\hat{k}_x^2(-\cos\delta+\frac{v^2}{c^2})+\hat{k}_y^2\frac{v^2}{c^2})\times\\ \cos\{r[k_x(1-\cos\delta)+k_y\sin\delta]-\omega\gamma(\tau_2-\tau_1)\},\end{aligned} \tag{20}$$

where

$$\hat{k_x}=\frac{k_x}{k},\quad \hat{k_y}=\frac{k_y}{k},\quad \hat{k_z}=\frac{k_z}{k}. \tag{21}$$

The integrand of the integral can be simplified by the variable change:

$$\begin{aligned}\hat{k}'_x=\hat{k}_x\cos\frac{\delta}{2}-\hat{k}_y\sin\frac{\delta}{2}\\ \hat{k}'_y=\hat{k}_x\sin\frac{\delta}{2}+\hat{k}_y\cos\frac{\delta}{2}.\end{aligned} \tag{22}$$

The terms which are odd in k_x vanish. Finally the correlation function takes the form:

$$\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle = \int d^3k h_0^2(\omega)\gamma^2\frac{1}{2}\times$$
$$(\cos\delta - \hat{k}_y 2\frac{v}{c}\cos\frac{\delta}{2} + +\hat{k}_x^2(-\cos^2\frac{\delta}{2}+\frac{v^2}{c^2}) + \hat{k}_y^2(\sin^2\frac{\delta}{2}+\frac{v^2}{c^2}))\times$$
$$\cos(2kr\sin\frac{\delta}{2}\hat{k}_y - ck(t_2-t_1)). \tag{23}$$

We have omitted primes in this expression.

2.4 General expressions for other correlation functions.

Similar expressions can be obtained for other CFs:

$$\langle E_2(\mu_1|A_1^{\mu_1},\tau_1)\, E_2(\mu_2|A_2^{\mu_2},\tau_2)\rangle = \langle E_2(\mu_1|0,0,0,\tau_1)\, E_2(\mu_1|0,0,0,\tau_2)\rangle =$$
$$\int d^3k\, h_0^2(\omega)\,\frac{1}{2}\times\;\frac{1}{2}[\,\hat{k}_x^2-\hat{k}_y^2\;+\;(\,1+\hat{k}_z^2\,)\cos\delta]\;\times\cos(2kr\sin\frac{\delta}{2}\hat{k}_y - ck(t_2-t_1)). \tag{24}$$

The third diagonal element of the electrical part of the CF is

$$\langle E_3(\mu_1|A_1^{\mu_1},\tau_1)\, E_3(\mu_2|A_2^{\mu_2},\tau_2)\rangle = \langle E_3(\mu_1|0,0,0,\tau_1)\, E_3(\mu_1|0,0,0,\tau_2)\rangle =$$
$$\int d^3k\, h_0^2(\omega)\,\frac{1}{2}\times\gamma^2\{\,1+\frac{v^2}{c^2}\cos\delta + \hat{k}_y(-2\frac{v}{c}\cos\frac{\delta}{2}) + \frac{v^2}{c^2}(-\hat{k}_x^2\cos^2\frac{\delta}{2}+\hat{k}_y^2\sin^2\frac{\delta}{2}) - \hat{k}_z^2\}\;\times$$
$$\cos(2kr\sin\frac{\delta}{2}\hat{k}_y - ck(t_2-t_1)). \tag{25}$$

The non-diagonal elements of the electrical components of the CFs are as follows:

$$\langle E_1(\mu_1|A_1^{\mu_1},\tau_1)\, E_2(\mu_2|A_2^{\mu_2},\tau_2)\rangle = \langle E_1(\mu_1|0,0,0,\tau_1)\, E_2(\mu_1|0,0,0,\tau_2)\rangle =$$
$$\int d^3k\, h_0^2(\omega)\,\frac{1}{2}\;\times\{\,-(1+\hat{k}_z^2)\,\frac{\gamma}{2}\sin\delta + \hat{k}_y\,\gamma\frac{v}{c}\sin\frac{\delta}{2}\}\;\times\cos(2kr\sin\frac{\delta}{2}\hat{k}_y - ck(t_2-t_1)). \tag{26}$$

It is easy to see that

$$\langle E_1(\mu_1|0,0,0,\tau_1)\, E_2(\mu_1|0,0,0,\tau_2)\rangle = -\langle E_2(\mu_1|0,0,0,\tau_1)\, E_1(\mu_1|0,0,0,\tau_2)\rangle. \tag{27}$$

The other non-diagonal elements of the CF with electric field components are zeroes :

$$\langle E_1(\mu_1|0,0,0,\tau_1)\, E_3(\mu_1|0,0,0,\tau_2)\rangle = \langle E_3(\mu_1|0,0,0,\tau_1)\, E_1(\mu_1|0,0,0,\tau_2)\rangle =$$
$$\langle E_2(\mu_1|0,0,0,\tau_1)\, E_3(\mu_1|0,0,0,\tau_2)\rangle = \langle E_3(\mu_3|0,0,0,\tau_1)\, E_2(\mu_1|0,0,0,\tau_2)\rangle = 0. \tag{28}$$

The CFs with magnetic components are as follows:

$$\langle H_1(\mu_1|A_1^{\mu_1},\tau_1)\ H_1(\mu_2|A_2^{\mu_2},\tau_2)\rangle = \langle H_1(\mu_1|0,0,0,\tau_1)\ H_1(\mu_1|0,0,0,\tau_2)\rangle =$$

$$\int d^3k\ h_0^2(\omega)\ \frac{1}{2}\ \times\gamma^2\times\{\ \hat{k}_y(\cos^2\delta - 2\frac{v}{c}\cos\frac{\delta}{2}) + \hat{k}_x^2(-\frac{1}{2}\sin^2\delta) + \hat{k}_y^2(\frac{1}{2}\sin^2\delta) + \hat{k}_z^2\cos\delta) +$$

$$\hat{k}_y\hat{k}_z^2(-\cos\frac{\delta}{2}\cos\delta) + \hat{k}_x^2\hat{k}_z^2\sin^2\frac{\delta}{2}\cos\delta + \hat{k}_y^2\hat{k}_z^2\cos^2\frac{\delta}{2}\cos\delta\}\times$$

$$\cos\,(2kr\sin\frac{\delta}{2}\hat{k}_y - ck(t_2-t_1)). \qquad (29)$$

So all CFs can be given as 3-dimensional integrals over (k,θ,ϕ). In the next subsection an example of calculation of these integrals is given.

2.5 Integral calculations: final expression for $E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle$.

All non zero expressions for CFs have a common integral over k. It can be easily calculated:

$$\int_0^\infty dkk^3\cos\{k(2r\sin\frac{\delta}{2}\sin\theta\sin\phi - c(t_2-t_1))\} = \frac{6}{\{2r\sin\frac{\delta}{2}\sin\theta\sin\phi - c(t_2-t_1)\}^4} =$$

$$= \frac{6}{[c(t_2-t_1)]^4}\frac{1}{[1-\frac{v}{c}\frac{\sin\delta/2}{\delta/2}\sin\theta\sin\phi]^4}. \qquad (30)$$

The integrals over θ and ϕ can be represented in terms of elementary functions. Let us show it for $\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_1|0,0,0,\tau_2)\rangle$

$$\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_1|0,0,0,\tau_2)\rangle = \frac{3\hbar c}{2\pi^2[c(t_2-t_1)]^4}\gamma^2\int_0^\pi d\theta$$

$$\times\{(\cos\delta\sin\theta + (-\cos^2\frac{\delta}{2} + \frac{v^2}{c^2})\sin^3\theta)\int_0^{2\pi} d\phi\ \frac{1}{(1+b\sin\phi)^4}$$

$$+(-2\frac{v}{c}\cos\frac{\delta}{2})sin^2\theta\int_0^{2\pi} d\phi\ \frac{\sin\phi}{(1+b\sin\phi)^4} + \sin^3\theta\int_0^{2\pi} d\phi\ \frac{\sin^2\phi}{(1+b\sin\phi)^4}\}, \qquad (31)$$

where $b\equiv k\ \sin\theta,\ \ k\equiv -\frac{v}{c}\frac{\sin\delta/2}{\delta/2}$. So k is a constant, not a wave vector.

We have used here:

$$\hat{k}_x = \sin\theta\cos\phi,\quad \hat{k}_y = \sin\theta\sin\phi,\quad \hat{k}_z = \cos\theta. \qquad (32)$$

The next step is to calculate the integral over ϕ. Because [8]

$$\int_0^{2\pi} d\phi\frac{1}{(1+b\sin\phi)^4} = \frac{\pi(2+3b^2)}{(1-b^2)^{7/2}}, \qquad (33)$$

$$\int_0^{2\pi} d\phi\frac{\sin\phi}{(1+b\sin\phi)^4} = \frac{-b\pi(4+b^2)}{(1-b^2)^{7/2}}, \qquad (34)$$

and

$$\int_0^{2\pi} d\phi \frac{\sin^2\phi}{(1+b\sin\phi)^4} = \frac{\pi(1+4b^2)}{(1-b^2)^{7/2}}, \tag{35}$$

the correlation function takes the form:

$$\begin{aligned}\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_1|0,0,0,\tau_2)\rangle &= \frac{3\hbar c}{2\pi^2[c(t_2-t_1)]^4}\gamma^2\{+[2\pi\cos\delta]\int_0^\pi d\theta\frac{\sin\theta}{(1-k^2\sin^2\theta)^{7/2}}\\ &+[3\pi k^2\cos\delta - 2\pi\cos^2(\delta/2) + 2\pi\beta^2 - 8\pi\beta k\cos(\delta/2) + \pi]\int_0^\pi d\theta\frac{\sin^3\theta}{(1-k^2\sin^2\theta)^{7/2}}\\ &+[-3\pi k^2\cos^2(\delta/2) + 3\pi\beta^2k^2 - 2\pi\beta k^3\cos(\delta/2) + 4\pi k^2]\int_0^\pi d\theta\frac{\sin^5\theta}{(1-k^2\sin^2\theta)^{7/2}}\},\end{aligned} \tag{36}$$

where ([21], 1.5.23, 1.2.43:

$$\int_0^\pi d\theta\frac{\sin\theta}{(1-k^2\sin^2\theta)^{7/2}} = \frac{2}{5(1-k^2)} + \frac{8}{15(1-k^2)^2} + \frac{16}{15(1-k^2)^3}, \tag{37}$$

$$\int_0^\pi d\theta\frac{\sin^3\theta}{(1-k^2\sin^2\theta)^{7/2}} = \frac{4}{15(1-k^2)^2} + \frac{16}{15(1-k^2)^3}, \tag{38}$$

$$\int_0^\pi d\theta\frac{\sin^5\theta}{(1-k^2\sin^2\theta)^{7/2}} = \frac{16}{15(1-k^2)^3}. \tag{39}$$

2.6 The Features of the $E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle$.

The correlation function depends on the difference $|\tau_2-\tau_1|$, a reasonable property of a correlation function, and parameters Ω, $\beta=\frac{\Omega r}{c}$, and k. The parameter k depends on δ according to $k=-\frac{v}{c}\frac{\sin(\delta/2)}{\delta/2}$. The δ is the angle the detector has rotated for the time t_2-t_1.

If $\delta\to 0$ then $k=-\beta$, and

$$\begin{aligned}\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_1|0,0,0,\tau_1\pm 0)\rangle &= \frac{3\hbar c}{2\pi^2[c(t_2-t_1)]^4}\gamma^2\{+2\pi[\frac{2}{5(1-\beta^2)} + \frac{8}{15(1-\beta^2)^2} + \frac{16}{15(1-\beta^2)^3}]\\ &+(-3\pi\beta^2-\pi)[\frac{4}{15(1-\beta^2)^2} + \frac{16}{15(1-\beta^2)^3}] + (\pi\beta^2+\pi\beta^4)\frac{16}{15(1-\beta^2)^3}\}.\end{aligned} \tag{40}$$

When $\beta\to 0$ then the value of this function is

$$\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_1|0,0,0,\tau_1\pm 0)\rangle_{\beta\to 0} = \frac{4\hbar c}{\pi[c(t_2-t_1)]^4}, \tag{41}$$

and it is positively defined as it is supposed to be for the quantity which is a contribution to the energy density of the electromagnetic field. The function is divergent when $t_1\to t_2$.

The expressions (37) and (38) have been obtained based on the [21], 1.2.43, 1.5.23 and (39).

In the next section we will show that the CFs should be modified to take its periodicity into consideration.

3 The Spectrum of the random classical zero-point electromagnetic radiation observed by a rotating detector.

3.1 Periodicity of the correlation function and Abel-Plana formula.

Later we will see that in the quantum case the Bogolubov coefficients are periodic due to the periodic motion of the rotating detector. We can expect that in a classical case periodic motion of a rotating detector should also result in periodicity of its measurements and particularly in the correlation function. Mathematically it means that $\langle E_1(\mu_1|0,t_1)E_1(\mu_2|0,t_2)\rangle = \langle E_1(\mu_1|0,t_1)E_1(\mu_{2n}|0,t_2+\frac{2\pi}{\Omega}n)\rangle$. Here $\Omega = \frac{2\pi}{T}$ is an angular velocity of the rotating detector and $n = \pm 0, 1, 2, 3, ...$ It is easy to show that (23) is periodic if $\omega = \Omega n$. It means that the rotating detector observes the random electromagnetic radiation with the same discrete spectrum as a rotating electrical charge radiates [18](39.29). Let us consider other consequences of the periodicity.

The equations (2) for the discrete spectrum should be modified to:

$$\vec{E}(\vec{r},t) = a\sum_{n=0}^{\infty}\sum_{\lambda=1}^{2}\int do\, k_n^2\,\hat{\epsilon}(\hat{k},\lambda)\,h_0(\omega_n)\ \cos[\vec{k_n}\vec{r} - \omega_n t - \Theta(\vec{k_n},\lambda)],$$
$$\vec{H}(\vec{r},t) = a\sum_{n=0}^{\infty}\sum_{\lambda=1}^{2}\int do\, k_n^2\,[\hat{k},\hat{\epsilon}(\hat{k},\lambda)]\,h_0(\omega_n)\ \cos[\vec{k_n}\vec{r} - \omega_n t - \Theta(\vec{k_n},\lambda)],$$
$$\vec{k_n} = k_n\hat{k},\quad k_n = k_0\,n,\quad k_0 = \frac{\Omega}{c},\quad \omega_n = c\,k_n,\quad do = d\theta\,d\phi\,\sin\theta,$$
$$\hat{k} = (\hat{k}_x,\hat{k}_y,\hat{k}_z) = (\sin\theta\,\cos\phi,\,\sin\theta\,\sin\phi,\,\cos\theta\,),\quad a = c\Omega. \tag{42}$$

The unit vector $\hat{k}$ defines a direction of the wave vector and does not depend on its value, n. The correlation function (17) takes the form:

$$\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle = a^2\langle\sum_{n_1,n_2=0}^{\infty}\ \sum_{\lambda_1,\lambda_2=1}^{2}\int do_1\,do_2\,h_0(\omega_{n_1})\,h_0(\omega_{n_2})\gamma^2\times$$
$$\{\hat{\epsilon}_{1x}(\hat{k}_1\lambda_1)\cos\delta + \hat{\epsilon}_{1y}(\hat{k}_1\lambda_1)(-\sin\delta) + (\hat{k}_{1x}\hat{\epsilon}_{1y}(\hat{k}_1\lambda_1) - \hat{k}_{1y}\hat{\epsilon}_{1x}(\hat{k}_1\lambda_1))\frac{v}{c}\}\times$$
$$\{\hat{\epsilon}_{2x}(\hat{k}_2\lambda_2) + (\hat{k}_{2x}\hat{\epsilon}_{2y}(\hat{k}_2\lambda_2) - \hat{k}_{2y}\hat{\epsilon}_{2x}(\hat{k}_2\lambda_2))\frac{v}{c}\}\times$$
$$\cos\{k_{n_1}\,[\hat{k_{1x}}[-r(1-\cos\delta)] + \hat{k_{1y}}(-r\sin\delta) - c\gamma\tau_1] - \theta(\hat{k_1}\lambda_1)\}\cos\{-k_{n_2}c\,\gamma\tau_2 - \theta(\vec{k_2}\lambda_2)\}\rangle. \tag{43}$$

In spherical coordinates the right side of the relation (18) should be changed [10], p.656:

$$\langle\cos\theta(\vec{k}_1\lambda_1)\cos\theta(\vec{k}_2\lambda_2)\rangle = \langle\sin\theta(\vec{k}_1\lambda_1)\sin\theta(\vec{k}_2\lambda_2)\rangle = \frac{1}{2}\delta_{\lambda_1\lambda_2}\delta^3(\vec{k}_1-\vec{k}_2) = \frac{1}{2}\delta_{\lambda_1\,\lambda_2}\,\frac{2}{k_1^2}\,\delta(k_1-k_2)\delta(\hat{k}_1-\hat{k}_2). \tag{44}$$

In the case of the discrete spectrum it takes the form:

$$\langle \cos\theta(\vec{k}_{n_1}\lambda_1)\cos\theta(\vec{k}_{n_2}\lambda_2)\rangle = \langle \sin\theta(\vec{k}_{n_1}\lambda_1)\sin\theta(\vec{k}_{n_2}\lambda_2)\rangle = \frac{1}{2}\delta_{\lambda_1\,\lambda_2}\,\frac{2}{k_0(k_0 n_1)^2}\,\delta_{n_1\,n_2}\delta(\hat{k}_1 - \hat{k}_2). \qquad (45)$$

The equation (19) $\sum_{\lambda=1}^{2}\epsilon_i(\vec{k}\lambda)\epsilon_j(\vec{k}\lambda) = \delta_{ij} - \hat{k}_i\hat{k}_j$ does not depend on n.

Then the correlation function (43) becomes

$$\langle E_1(\mu_1|0,0,0,\tau_1)E_1(\mu_2|0,0,0,\tau_2)\rangle = a^2\int do\gamma^2\{\cos\delta - \hat{k}_y\frac{2v}{c}\cos\frac{\delta}{2} + \hat{k}_x^2(-\cos^2\frac{\delta}{2} + \frac{v^2}{c^2}) + \hat{k}_y^2(\sin^2\frac{\delta}{2} + \frac{v^2}{c^2})\}$$
$$\times\sum_{n=0}^{\infty}(k_0\,n)^2\,h_0^2(ck_0 n)\,\cos[k_0 n\,(\,2r\sin\frac{\delta}{2}\hat{k}_y - c(t_2 - t_1)\,)] =$$
$$= \frac{a^2\,\gamma^2\,k_0^3 c\hbar}{2\pi^2}\int do\,[\cos\delta - \hat{k}_y 2\frac{v}{c}\cos\frac{\delta}{2} + \hat{k}_x^2(-\cos^2\frac{\delta}{2} + \frac{v^2}{c^2}) + \hat{k}_y^2(\sin^2\frac{\delta}{2} + \frac{v^2}{c^2})]\times\sum_{n=0}^{\infty}n^3\,\cos n\,F, \qquad (46)$$

where

$$F = k_0\,(\,2r\sin\frac{\delta}{2}\hat{k}_y - c(t_2 - t_1)\,) = \delta\,[1 - \frac{v}{c}\frac{\sin\delta/2}{\delta/2}\sin\theta\sin\phi],\quad \delta = \Omega(t_2 - t_1). \qquad (47)$$

The sum over n in this equation

$$S \equiv \sum_{n=0}^{\infty}n^3\cos(n\,F) \qquad (48)$$

can be evaluated using the Abel-Plana summation formula [3],[**?**], [11]:

$$\sum_{n=0}^{\infty}f(n) = \int_0^{\infty}f(x)\,dx + \frac{f(0)}{2} + i\int_0^{\infty}dt\,\frac{f(it) - f(-it)}{e^{2\pi t} - 1}, \qquad (49)$$

Having utilized this formula and following to [14] we come to the following expression:

$$\Omega^4 S = \int_0^{\infty}d\,\omega\omega^3\cos(\omega\tilde{F}) + \int_0^{\infty}d\omega\frac{2\omega^3\cosh(\omega\tilde{F})}{e^{2\pi\omega/\Omega} - 1},\quad \tilde{F} = \tfrac{F}{\Omega}. \qquad (50)$$

The integrals in this expression can be computed and put in two forms:

$$S = \frac{6}{F^4} + [\frac{3 - 2\sin^2(F/2)}{8\sin^4(F/2)} - \frac{6}{F^4}] \qquad (51)$$

or

$$S = \frac{6}{F^4} + 6\sum_{n=1}^{\infty}\frac{1}{(2\pi n)^4}[\frac{1}{(1 + \frac{F}{(2\pi n)^4})} + \frac{1}{(1 - \frac{F}{(2\pi n)^4})}]. \qquad (52)$$

Complete calculation of the integrals over θ and ϕ in the (46) will be given elsewhere. In this article we wold like to focus on the expression (50) which allows a simple physical interpretation. It will be done in the next subsection.

3.2 Planck spectrum of the energy density of random classical electromagnetic radiation observed by a rotating detector.

Let us compare the expression (50)for S with the expression [1] (74) for the Fourie component of the spectral function $\frac{1}{2}\hbar\omega \coth \frac{\hbar\omega}{2kT}$ of the electromagnetic radiation with Planck's spectrum at the temperature T,with the zero-point radiation:

$$\frac{1}{2}\int_0^\infty d\omega\omega^3 \coth(\frac{\hbar\omega}{2kT}) \cos\omega t = \frac{1}{2}\,[\int_0^\infty d\omega\omega^3 \cos\omega t + \int_0^\infty d\omega \frac{2\omega^3}{e^{\frac{\hbar\omega}{kT}}-1} \cos\omega t\,]. \tag{53}$$

The right sides of these expressions are very similar, except for two features: in the first expression $\tilde{F}$ and *cosh* are used instead of t and *cos* respectively in (53) . For $\tilde{F} = 0$ and $t = 0$ though the right sides of both expressions are identical if we define a new variable T_{rot} according to:

$$T_{rot} = \frac{\hbar\Omega}{2\pi k}, \tag{54}$$

where k is a Bolzman constant.

This remarkable resemblance brings up the idea that the energy density of the random classical electromagnetic radiation measured by a detector, rotating through a zero point radiation, has the Planck spectrum at the temperature T_{rot} (54).

Using the technique, described above for a discrete spectrum, the energy density

$$w(\mu) = \frac{1}{8\pi}\langle\sum_{i=1}^{3}(E_i^2(\mu|) + H_i^2(\mu|))\rangle = \frac{1}{4\pi}\{[E_1^2(\lambda) + E_3^2(\lambda)]\gamma^2(1+\beta^2) + E_2^2(\lambda)\}, \tag{55}$$

measured by the rotating observer at an instantaneous inertial reference frame μ, can be given in the form:

$$w(\mu) = \frac{2(4\gamma^2-1)}{3}\,\frac{\hbar}{c^3\pi^2}\,\frac{1}{2}\Omega^4 \sum_{n=0}^{\infty} n^3 \tag{56}$$

or with the help of (50) for $F = 0$ as

$$w(\mu) = \frac{2(4\gamma^2-1)}{3}\,w(T_{rot}), \tag{57}$$

where

$$w(T_{rot}) = \frac{\hbar}{c^3\pi^2}\,\frac{1}{2}(\int_0^\infty d\,\omega\omega^3 + \int_0^\infty d\omega \frac{2\omega^3}{e^{\hbar\omega/kT_{rot}}-1}\,). \tag{58}$$

is the full (with a zero point radiation included) averaged energy density measured by an inertial observer at the temperature T_{rot}. The $w(\mu)$ does not depend on time and we omitted the index t (or

τ). In the limiting case of $\Omega \to 0$, $T_{rot} \to 0$, $\gamma \to 1$, and $\langle w(\mu)\rangle = \langle w(T_{rot} = 0)\rangle$. In the relationship (57) $\langle w(T_{rot})$ is divergent for any T_{rot}.

The energy density (58)has two terms and it is the first term which is divergent. The second one is connected with periodicity of the detector rotation. It is convergent. Usually such convergent term is referred to as a regularized energy density $reg\ w(\mu)$ [[14]], p.969 and considered as an observable physical quantity. It is equal to

$$reg\ w(\mu) = \frac{2(4\gamma^2 - 1)}{3}\, w_{rad}, \tag{59}$$

where

$$w_{rad} = 4\, \frac{\pi^2 k^4}{60(c\hbar)^3}\, T_{rot}^4. \tag{60}$$

It is well known expression of the energy density of the black radiation at the temperature T_{rot} [[29], (60,14)]:

$$w_{rad} = \frac{4\sigma}{c} T_{rot}^4, \tag{61}$$

and k is a Bolzman constant, σ is a Stephan-Bolzman constant , and w_{rad} is the density of the energy of black radiation, without zero point radiation, at the temperature T_{rot}.

So, due to periodicity of the motion, an observer rotating through a zero point radiation should see the energy density, which would have been observed by an observer moving in a thermal bath at the temperature $T_{rot} = \frac{\hbar\Omega}{2\pi k}$, and multiplied by the factor $\frac{2}{3}(4\gamma^2 - 1)$. This factor comes from integration in (46) over angles and therefore is a consequence of anisotropy of the electromagnetic field measured by an observer with velocity β. When the angular velocity of the detector is zero, the regularized energy density is zero as well. For a fixed angular velocity, the energy density depends on a radius of the detector circular path via γ^2. When r and therefore γ increases the regularized energy density increases as well.

4 Massless scalar field. Correlation function at a rotating detector.

4.1 Classical consideration.

The calculation of the correlation function for a massless scalar field is much simpler than the calculation in the electromagnetic field case because the scalar field does not change under Lorentz trans-

formations. The correlation function measured by an observer rotating through a classical massless zero-point scalar field radiation has the form:

$$\langle \psi_s(\mu_1|A_1^{\mu_1}, t_1^{\mu_1})\psi_s(\mu_2|A_2^{\mu_2}, t_2^{\mu_2})\rangle = \langle \psi_s(\lambda_1|A_1^{\lambda_1}, t_1^{\lambda_1})\psi_s(\lambda_2|A_2^{\lambda_2}, t_2^{\lambda_2})\rangle =$$
$$\langle \psi_s(\lambda_1'|A_1^{\lambda_1'}, t_1^{\lambda_1'})\psi_s(\lambda_2|A_2^{\lambda_2}, t_2^{\lambda_2})\rangle = \langle \psi_s(\lambda_2|A_1^{\lambda_2}, t_1^{\lambda_2})\psi_s(\lambda_2|A_2^{\lambda_2}, t_2^{\lambda_2})\rangle, \tag{62}$$

where $(A_1^{\mu_1}, t_1^{\mu_1})$, $(A_1^{\lambda_1}, t_1^{\lambda_1})$, $(A_1^{\lambda_1'}, t_1^{\lambda_1'})$, and $(A_1^{\lambda_2}, t_1^{\lambda_2})$ are 4-coordinates of the rotating detector at the first position, taken in the reference frames μ_1, λ_1, λ_1', and λ_2 respectively. Transitions between these reference frames were discussed in a previous section. In the last expression of the equation, all coordinates are defined in the same reference frame λ_2. Then taking into consideration (14) we can write:

$$\psi_s(\lambda_2|A_1^{\lambda_2}, t_1^{\lambda_2}) = \int d^3k_1 f(\omega_1)\cos\{-k_{1x}r(1-\cos\delta) - k_{1y}r\sin\delta - \omega_1\tau_1\gamma - \theta(k_1)\} \tag{63}$$

$$\psi_s(\lambda_2|A_2^{\lambda_2}, t_2^{\lambda_2}) = \int d^3k_2 f(\omega_2)\cos\{-\omega_2\tau_2\gamma - \theta(k_2)\} \tag{64}$$

Using these expressions and [1]

$$\langle \cos\theta(\vec{k}_1)\cos\theta(\vec{k}_2)\rangle = \langle \sin\theta(\vec{k}_1)\sin\theta(\vec{k}_2)\rangle = \frac{1}{2}\delta^3(\vec{k}_1 - \vec{k}_2), \quad f^2(\omega) = \frac{\hbar c^2}{2\pi^2\omega}, \tag{65}$$

we come to the expression:

$$\langle \psi_s(\mu_1|A_1^{\mu_1}, t_1^{\mu_1})\psi_s(\mu_2|A_2^{\mu_2}, t_2^{\mu_2})\rangle = \int d^3k f^2(\omega)\frac{1}{2}\cos\{r(k_x(1-\cos\delta) + k_y\sin\delta) - \omega\gamma(\tau_2 - \tau_1)\} \tag{66}$$

or, after coordinate change (22) in the integrand and omitting primes, to:

$$\langle \psi_s(\mu_1|0,0,0,\tau_1)\psi_s(\mu_2|0,0,0,\tau_2)\rangle = \int d^3k \times \frac{\hbar c^2}{2\pi^2\omega} \times \frac{1}{2} \times \cos(2rk_y\sin\frac{\delta}{2} - ck\gamma(\tau_2 - \tau_1)). \tag{67}$$

Having integrated it over k, the right side takes the form:

$$-\frac{\hbar c}{4\pi^2}\int_0^\pi d\theta\,\sin\theta\int_0^{2\pi} d\phi\,\,[E\,\sin\phi - B]^{-2}. \tag{68}$$

where $B = \gamma\tau c$, $E = 2r\sin\theta\,\sin\frac{\Omega\gamma\tau}{2}$, and $\tau = \tau_2 - \tau_1$.

Because $B - |E| = c\gamma\tau\{1 - \frac{v}{c}|\sin\theta\,\frac{\sin\pi(\gamma\tau/T)}{\pi(\gamma\tau/T)}|\} > c\gamma\tau(1 - v/c) > 0$, and using [21] we obtain :

$$\int_0^{2\pi} d\phi\,\frac{1}{[E\,\sin\phi - B]^2} = \frac{2\pi B}{(B^2 - E^2)^{3/2}}. \tag{69}$$

Having integrated over θ we come to the final expression of the CF for the correlation function of the random classical massless scalar field at the rotating detector moving through a zero point fluctuating massless scalar radiation:

$$\langle \psi_s(\mu_1|0,0,0,\tau_1)\psi_s(\mu_2|0,0,0,\tau_2)\rangle = -\frac{\hbar c}{\pi}\frac{1}{(\gamma(\tau_2-\tau_1)c)^2 - 4r^2\sin^2\frac{\Omega\gamma(\tau_2-\tau_1)}{2}}. \tag{70}$$

This correlation function received in the classical approach based on two references systems μ_τ and λ_τ is identical, up to a constant factor, to the Wightman function [9], [22](3.59) received in the quantum case.

The physical sense of this function and its Fourie component has been investigated by several authors in the frame of a quantum theory, starting from Pfautsch[[25]]. Davis, Dray, and Manogue [[9]] think that the spectrum found in [[25]] numerically is only "a reminiscent of a Planck spectrum" because "the Bogolubov transformation between rotating and non rotating modes is trivial" and two "sets of modes are identical". Recently De Lorency, De Paola, and Svaiter [[23]] using a proper mapping between rotating and non rotating coordinate systems found new modes of the scalar filed in the rotating system and showed that the Bogolubov transformation is not zero. But this mapping has very unusual features. So the question about non zero Bogolubov transformation is still open for further investigation.

Below we investigate this issue again both in classical, with some natural periodicity condition, and quantum approach calculating the Bogolubov transformation. Our consideration is based on two reference systems and does not use mapping between rotating and nonrotating coordinate systems.

4.2 Quantum consideration. Bogolubov transformation between modes of a massless scalar field in a rotating reference system and the laboratory coordinate system.

Usually the question if the vacuums of massless scalar field observed by a rotating observer and inertial one are unitary equivalent is discussed using Bogolubov transformations between inertial (laboratory) reference system and a rotating reference system. The history of this issue is given in [23]. It was found that the Bogolubov coefficients are null if rotating and inertial coordinates are mapped as

$$t = t', \quad r = r', \quad \theta = \theta' - \Omega t', z = z'.$$

The significant feature of this coordinate system is that an attendant rotating reference frame is co-moving with the observer. In the attendant rotating reference frame the observer is permanently at rest.

Recently Lorenci, de Paola, and Svaiter [23] have shown that Trocheries -Takeno coordinates with non linear connection between a linear and angular velocities of the observer

$$t = t' \cosh \Omega r' - r'\theta' \sinh \Omega r', \;\; r = r', \;\; \theta = \theta' \cosh \Omega r' - \frac{t'}{r'} \sinh \Omega r', \;\; z = z'$$

should be used to get non zero Bogolubov coefficients.

In this case the location of the observer, $(R_0, \theta', z' = 0)$ is constant in the rotating reference frame. But the observer and the reference frame are not co-moving because the metrics of the rotating reference frame depends on time and a distance between any point and the observer changes in time as well. The rotating reference frame used in [23] is not a rigid one in the sense defined in [17].

The rotating reference system $\{\mu_t\}$ defined in this work is also not co-moving with the rotating detector. It consists of infinite number of inertial reference frames μ_t moving in the flat space-time of the laboratory system. They do not accompany the observer. The frame μ_t labelled by t agrees with the observer only once , instantaneously, at the respective moment of time t. No special coordinates are used in these frames and they have the Minkovsky metric. The reference system $\{\mu_t\}$ can be used for both classical and quantum systems. We will see that the reference system $\{\mu_\tau\}$ is very useful to calculate Bogolubov's coefficients. The final expression of the coefficients is given in terms of elementary functions. It is much simpler than the expression received in [23] and explicitly not zero.

Let us consider the quantized scalar massless scalar field in two reference frames μ_t and λ. The λ reference frame agrees with μ_t with t=0.

In both global inertial reference frames, μ_t and λ, the Fourier series for the operators of scalar field have similar forms [27]

$$\begin{aligned}\psi(\vec{x}, t) &= \int d^3k[a(k) f_k(\vec{x}, t) + a^+(k) f^*(\vec{x}, t)],\\ \psi^{\mu_t}(\xi^{\vec{\mu}_t}, \eta^{\mu_t}) &= \int d^3k[a_k^{\mu_t} f_k^{\mu_t}(\xi^{\vec{\mu}_t}, \eta^{\mu_t}) + a^{\mu_t +} f_k^{\mu_t *}(\xi^{\vec{\mu}_t}, \eta^{\mu_t})].\end{aligned} \tag{71}$$

For simplicity purposes we have omitted the index λ in the first expression. The operator $\psi(\vec{x}, t)$ describes the quantized scalar field in the laboratory inertial reference system at the time t. The

operator $\psi^{\mu_t}()$ describes the same quantized scalar field in the inertial reference frame μ_t which is instantaneously at rest relative to the detector at the *same* laboratory time t. There is a close relationship between the coordinates $\vec{x}, t$ in the laboratory reference system λ and $\vec{\xi}^{\mu_t}, \eta^{\mu_t}$ of the inertial reference frame μ_t. It is defined below. The development of the ψ operator in time in the laboratory system corresponds to the description of the operator ψ^{μ_t} in a *system* of inertial reference frames $\{\mu_t\}$, *one instantaneous reference frame for each moment of time.*

The plane waves have similar forms [27],(9.6):

$$f_k(x,t) = \frac{1}{((2\,\pi)^3\,2\omega_k)^{1/2}}\exp\{-i\,(\omega_k\,t - \vec{k}\,\vec{x})\,\},$$
$$f_k^{\mu_t}(\,\vec{\xi}^{\mu_t},\,\eta^{\mu_t}\,) = \frac{1}{(\,(2\,\pi)^3\,2\,\omega_k\,)^{1/2}}\exp\{-i(\,\omega_k\,\eta^{\mu_t} - \vec{k}\,\vec{\xi}^{\mu_t}\,)\}. \tag{72}$$

We assume that operators $a(k)$ and a^{μ_t} are different in both reference frames and have to find the connection between them. For scalar field we have:

$$\psi(\vec{x},t) = \psi^{\mu_t}(\vec{\xi}^{\mu_t},\,\eta^{\mu_t}) \tag{73}$$

Here spatial coordinates $\vec{\xi}^{\mu_t}$ and time coordinate η^{μ_t} are considered as functions of $\vec{x}$ and t. In our case:

$$\xi_1^{\mu_t} = x_1\cos\delta_t + x_2\sin\delta_t - 2r\sin^2\frac{\delta_t}{2},$$
$$\xi_2^{\mu_t} = x_1(-\gamma\sin\delta_t) + x_2\gamma\cos\delta_t + t(-v\gamma) - (r\sin\delta_t + a_\tau)\gamma,$$
$$\xi_3^{\mu_t} = x_3,$$
$$\eta^{\mu_t} = x_1(\frac{v}{c^2}\gamma\sin\delta_t) + x_2(-\frac{v}{c^2}\gamma\cos\delta_t) + t\gamma + (r\sin\delta_t + a_\tau)\frac{v}{c^2}\gamma. \tag{74}$$

These relationships have been obtained by three consequent transformations. The first is a Lorentz transformation between μ_t reference frame and λ_t. It was described above in terms of μ_τ and λ_τ. The second one is a rotation from λ_t to λ'_t. The λ'_t reference frame is parallel to the λ reference frame. And the third one is a shift from from λ'_t to λ.This shift is an opposite to the direction of the detector rotation. So ψ^{μ_t} depends on t in two ways and such dependence on time finally makes the Bogolubov's coefficients non zero. First, the variables $\vec{\xi}^{\mu_t}$ and η^{μ_t} depend on t via the Lorentz transformation . And second, the rotation by the angle $\delta_t = \Omega t$, the parameter a_τ of the Lorentz transformation, and the shift depend on t as well.

Having constructed scalar products of (73) with $f_{k'}$

$$\int d^3k[a(k)(f_{k'}, f_k) + a^+(k)(f_{k'}, f_k^*)] = \int d^3k[a_k^{\mu_t}(f_{k'}, f_k^{\mu_t}) + a^{\mu_t+}(f_{k'}, f_k^{\mu_t*})] \tag{75}$$

and $f_{k'}^*$

$$\int d^3k[a(k)(f_{k'}^*, f_k) + a^+(k)(f_{k'}^*, f_k^*)] = \int d^3k[a_k^{\mu_t}(f_{k'}^*, f_k^{\mu_t}) + a^{\mu_t+}(f_{k'}^*, f_k^{\mu_t*})], \tag{76}$$

where a scalar product in the scalar field is defined according to [22], (2.9) [03.25.06, 37]:

$$(f_{k'}, f_k^{\mu_t}) = -i\int d^3[f_{k'}\frac{\partial(f^{\mu_t})^*}{\partial t} - \frac{\partial f_{k'}}{\partial t}(f^{\mu_t})^*] \tag{77}$$

and

$$(f_{k'}, f_k) = \delta^3(\vec{k} - \vec{k'}), \quad f_{k'}^*, f_k^*) = -\delta^3(\vec{k} - \vec{k'}), \quad f_{k'}^*, f_k) = f_{k'}, f_k^*) = 0, \tag{78}$$

we arrive at the following relationships between a_k and $a_k^{\mu^t}$:

$$\begin{aligned} a_{k'} &= \int d^3k\ [a_k^{\mu_t}\alpha_{kk'}^* + a_k^{+\mu_t}\beta_{kk'}], \\ a_{k'}^+ &= \int d^3k\ [a_k^{\mu_t}\beta_{kk'}^* + a_k^{+\mu_t}\alpha_{kk'}]. \end{aligned} \tag{79}$$

The Bogolubov coefficients $\beta_{kk'}$ and $\alpha_{kk'}$ are defined in our notations as follows:

$$\begin{aligned} \beta_{kk'} &= (f_{k'}, f_k^{*\mu_t}), \\ \alpha_{kk'} &= -(f_{k'}^*, f_k^{*\mu_t}). \end{aligned} \tag{80}$$

The most interesting for us is $\beta_{kk'}$. Taking into consideration (72) $\beta_{kk'}$ can be given in the form:

$$\beta_{kk'} = -i\int d^3x f_{k'} f_k^{\mu_t}[-i\omega_k\frac{\partial\eta^{\mu_t}}{\partial t} + i\vec{k}\frac{\partial\vec{\xi}^{\mu_t}}{\partial t} + i\omega_{k'}]. \tag{81}$$

Using (74) and

$$\begin{aligned} \frac{\partial a_\tau}{\partial t} &= -v, \\ \frac{\partial \delta_t}{\partial t} &= \Omega, \end{aligned} \tag{82}$$

this expression finally equals to:

$$\begin{aligned} \beta_{kk'} = B_0B_1\int_{-\infty}^{\infty} dx_1 x_1 \exp(ix_1A_1)\int_{-\infty}^{\infty} dx_2 \exp(ix_2A_2)\int_{-\infty}^{\infty} dx_3 \exp(ix_3A_3) + \\ +B_0B_2\int_{-\infty}^{\infty} dx_1 \exp(ix_1A_1)\int_{-\infty}^{\infty} dx_2 x_2 \exp(ix_2A_2)\int_{-\infty}^{\infty} dx_3 \exp(ix_3A_3) + \\ +B_0B_3\int_{-\infty}^{\infty} dx_1 \exp(ix_1A_1)\int_{-\infty}^{\infty} dx_2 \exp(ix_2A_2)\int_{-\infty}^{\infty} dx_3 \exp(ix_3A_3), \end{aligned} \tag{83}$$

where

$$B_0 = \frac{-i}{(2\pi)^3 2(\omega_k \omega_{k'})^{1/2}} \exp\{-it[\omega_{k'} + \frac{\omega_k}{\gamma}] + ir[-k_1 + k_1 \cos\delta_t - (k_2 + \frac{v}{c^2}\omega_k)\gamma \sin\delta_t]\},$$

$$A_1 = k_1' + k_1 \cos\delta_t - (k_2 + \omega_k \frac{v}{c^2})\gamma \sin\delta_t,$$

$$A_2 = k_2' + k_1 \sin\delta_t + (k_2 + \omega_k \frac{v}{c^2})\gamma \cos\delta_t,$$

$$A_3 = k_3' + k_3,$$

$$B_1 = -i\Omega[k_1 \sin\delta_t + (k_2 + \omega_k \frac{v}{c^2})\gamma \cos\delta_t],$$

$$B_2 = -i\Omega[k_1 \cos\delta_t - (k_2 + \omega_k \frac{v}{c^2})\gamma \sin\delta_t],$$

$$B_3 = -ir\Omega[k_1 \sin\delta_t + (k_2 + \omega_k \frac{v}{c^2})\gamma \cos\delta_t] + i(\omega_{k'} - \frac{\omega_k}{\gamma}). \tag{84}$$

It is easy to show that the first two lines in the expression for $\beta_{kk'}$ are zeros because

$$\int_{-\infty}^{+\infty} dx x \exp\{ixA\} = 0. \tag{85}$$

Indeed, if $A = 0$ then $\int_{-\infty}^{+\infty} dx x = 0$. If $A \neq 0$ then

$$\int_{-\infty}^{+\infty} dx x \exp ixA = \lim_{\lambda\to 0}\{\int_0^{\infty} dx x \exp x(iA - \lambda) - \int_0^{\infty} dx x \exp -x(iA + \lambda\} =$$

$$\lim_{\lambda\to 0} \frac{(2\lambda)(2iA)}{\lambda^2 + A^2} = 0.$$

Besides

$$\int dx_i \exp(ix_i A_i) = 2\pi\delta(A_i), \quad i = 1, 2, 3.$$

Then we obtain:

$$\beta_{kk'} = \frac{-i}{2(\omega_k \omega_{k'})^{1/2}} \exp\{-it[\omega_{k'} + \frac{\omega_k}{\gamma}] + ir[-k_1 - k_1']\} \; [-ir\Omega(-k_2') + i(\omega_{k'} - \frac{\omega_k}{\gamma})] \times$$

$$\times\, \delta\, (k_1' + k_1 \cos\delta_t - (k_2 + \omega_k \frac{v}{c^2})\gamma \sin\delta_t) \;\; \delta\, (k_2' + k_1 \sin\delta_t + (k_2 + \omega_k \frac{v}{c^2})\gamma \cos\delta_t) \;\; \delta\, (k_3' + k_3) \tag{86}$$

We have taken into consideration that, because of the δ-function features, $\beta_{kk'} \neq 0$ only if

$$-\, k_1' = k_1 \cos\delta_t - (k_2 + \omega_k \frac{v}{c^2})\gamma \sin\delta_t$$

and

$$-\, k_2' = k_1 \sin\delta_t + (k_2 + \omega_k \frac{v}{c^2})\gamma \cos\delta_t$$

To interpret the physical sense of $\beta_{kk'}$ let us evaluate the expectation value of the operator of number of particles $N_{k'} = a^+_{k'} a_{k'}$ in the vacuum state $|0^{\mu_t}>$ that is $<0^{\mu_t}|N_{k'}|0^{\mu_t}>$. Obviously that $<0|N_{k'}|0>= 0$ because $a_k|0>= 0$ and $<0|a^+_k = 0$ by definition of the vacuum state $|0>$. It is easy to see using (79) that

$$<0^{\mu_t}|N_{k'}|0^{\mu_t}>= \int d^3\tilde{k}\beta^*_{\tilde{k}k'} \ <0^{\mu_t}|a^{\mu_t}_{\tilde{k}} \ | \int d^3k\beta_{kk'}a^{+\mu_t}_k|0^{\mu_t}> \tag{87}$$

because $a^{\mu_t}_k|0^{\mu_t}>= 0$ and $<0^{\mu_t}|a^{+\mu_t}_k = 0$. We can say that the vacuum $|0^{\mu_t}>$ of modes $f^{\mu_t}_k$ contains $<0^{\mu_t}|N_{k'}|0^{\mu_t}>$ particles of modes f_k [22] (3.42).

We need to compute the integral operator with the operator $a^{+\mu_t}(k)$ in its integrand

$$\hat{I}_{k'} = \int d^3k \ \beta_{kk'} \ a^{+\mu_t}(k) = \int \frac{d^3k}{\omega_k} \ \omega_k \ \beta_{kk'} \ a^{+\mu_t}(k). \tag{88}$$

We use here $a^{+\mu_t}(k)$ rather than $a^{+\mu_t}_k$.

To calculate the integral we have to change integrand variables to simplify arguments of the δ functions in (86). Let us use first the Lorentz transformation between $(\omega_k, k_1, k_2, k_3)$ and $(\omega_\kappa, \kappa_1, \kappa_2, \kappa_3)$:

$$\omega_k = \gamma(\omega_\kappa - v\kappa_2), \ \ k_2 = \gamma(\kappa_2 - \omega_\kappa \frac{v}{c^2}), \ \ k_1 = \kappa_1 \ \ k_3 = \kappa_3 \tag{89}$$

or

$$\omega_\kappa = \gamma(\omega_k + vk_2), \ \ \kappa_2 = \gamma(k_2 + \omega_k \frac{v}{c^2}), \ \ \kappa_1 = k_1, \ \ \kappa_3 = k_3. \tag{90}$$

Then the integral $\hat{I}_{k'}$ takes the form:

$$\hat{I}_{k'} = \int \frac{d^3\kappa}{\omega_\kappa} \ \frac{[\gamma(\omega_\kappa - v\kappa_2)]^{1/2} \ \ [v \ k'_2 + (\omega_{k'} - \frac{\gamma(\omega_\kappa - v\kappa_2)}{\gamma})]}{2(\omega_{k'})^{1/2}} \ G \times$$
$$\delta(k'_1 + \kappa_1 \cos\delta_t - \kappa_2 \sin\delta_t) \ \ \delta(k'_2 + \kappa_1 \sin\delta_t + \kappa_2 \cos\delta_t) \ \ \delta(k'_3 + \kappa_3) \ |a^{+\mu_t}(\kappa), \tag{91}$$

where

$$G = \exp\{-it[\omega_{k'} + \frac{\gamma(\omega_\kappa - v\kappa_2)}{\gamma}] + ir[-\kappa_1 - k'_1]\} \tag{92}$$

and we have used relationship

$$\frac{d^3k}{\omega_k} = \frac{d^3\kappa}{\omega_\kappa}, \tag{93}$$

which is a Lorentz transformation invariant [26], \$10.

The next variable changes in the integrand is the following rotation transformation between $(\omega_\kappa, \kappa_1, \kappa_2, \kappa_3)$ and $(\omega_k, k_1, k_2, k_3)$:

$$k_1 = \kappa_1 \cos\delta_t - \kappa_2 \sin\delta_t, \quad k_2 = \kappa_1 \sin\delta_t + \kappa_2 \cos\delta_t, \quad k_3 = \kappa_3, \quad \omega_k = \omega_\kappa, \tag{94}$$

or

$$\kappa_1 = k_1 \cos\delta_t + k_2 \sin\delta_t, \quad \kappa_2 = -k_1 \sin\delta_t + k_2 \cos\delta_t. \tag{95}$$

In these variables the integral $\hat{I}_{k'}$ and its integrand take the form:

$$\hat{I}_{k'} = \int \frac{d^3k}{\omega_k} \; \frac{\{\gamma[\omega_k - v(-k_1 \sin\delta_t + k_2 \cos\delta_t)]\}^{1/2} \; [\omega_{k'} - \omega_k + vk'_2 + v(-k_1 sin\delta_t + k_2 \cos\delta_t)]}{2\{\omega_{k'}\}^{1/2}} \; \tilde{G}$$
$$\delta(k_1 + k'_1) \;\; \delta(k_2 + k'_2) \;\; \delta(k_3 + k'_3) \; a^{\mu_t}(k), \tag{96}$$

with

$$\tilde{G} = \exp i\{t[-\omega_{k'} - \omega_k + v(-k_2 \sin\delta_t + k_2 \cos\delta_t)] - r[k'_1 + k_1 \cos\delta_t + k_2 \sin\delta_t]\}. \tag{97}$$

So the expressions in the $\delta-$ functions got very simple form.

Having integrated this expression and taking into consideration that

$$\omega_k = \omega_{|-\vec{k}'|} = \omega_{k'}, \tag{98}$$

we obtain:

$$\hat{I}_{k'} = \frac{v \; \gamma^{1/2}[\omega_{k'} - v(k'_1 \sin\delta_t - k'_2 \cos\delta_t)]^{1/2} \; [k'_2 + k'_1 \sin\delta_t - k'_2 \cos\delta_t]}{2\omega_{k'}^{3/2}} \times$$
$$\exp\{it[-2\omega_{k'} + v(k'_2 \sin\delta_t - k'_2 \cos\delta_t)] + ir[-k'_1 + k'_1 \cos\delta_t + k'_2 \sin\delta_t]\} a^{\mu_t}(-\vec{k}'). \tag{99}$$

In the same way we could show that

$$\int d^3k \beta^*_{kk'} a^{\mu_t}(k) = \hat{I}^+_{k'}. \tag{100}$$

Then because

$$< 0^{\mu_t}|a^{\mu_t}(-\vec{k}')|a^{+\mu_t}(-\vec{k}')|0^{\mu_t} >= 1, \tag{101}$$

$$< \; 0^{\mu_t} \; |N_{k\prime}| \; 0^{\mu_t} \; >= \hat{I}^+_{k'}|\hat{I}_{k'} = \; \frac{v^2 \; \gamma \; [\omega_{k'} - v(k'_1 \sin\delta_t - k'_2 \cos\delta_t)](k'_2 + k'_1 \sin\delta_t - k'_2 \cos\delta_t)^2}{4\omega^3_{k'}}. \tag{102}$$

So $< O^{\mu_t}|N_{k'}|O^{\mu_t} > \neq 0$. Following a usual interpretation [22] (3.41), the fact of non zero value of $\beta_{kk'}$ means that detector rotating in a vacuum state $| \, 0 \; >$ observes non zero number of particles of mode $f_k^{\mu_t}$. The vacuums $| \, 0 \; >$ and $| \, 0^{\mu_t} \; >$ and associated Fock spaces are not unitary equivalent. In the limit of $v \to 0$ or $\delta_t \to 0$ $\beta_{kk'} \to 0$, and both Fock spaces and their vacuums agree, what is supposed to be.

Specific feature of the Bogolubov transformation found here is its dependence on time. It is a consequence of the way a non inertial rotating detector observes the quantized massless scalar field in the vacuum state. At each moment the detector uses that inertial reference frame which is momentarily at rest relative to and agrees with it at that moment of time. There is another example of the time dependent Bogolubov transformation [28] \$6.2. The quantized Fermi field interacting with an external classical uniform electric field can be represented as a free field at any time if a vacuum state at that moment is redefined correspondingly. So an external classical field is turned off the same way as the inertial reference system is switched to the inertial reference frame in our case, by the redefining a vacuum state of the quantized filed. Our calculation of the Bogolubov coefficients was greatly motivated by that result.

5 The spectrum of the random classical massless scalar field observed by a rotating detector.

5.1 Periodicity and the correlation function.

Under the assumption about periodicity the correlation function of the massless scalar field at the rotating detector (62)becomes [12.15.04]:

$$\langle \psi_s(\mu_1|A_1^{\mu_1}, t_1^{\mu_1})\psi_s(\mu_2|A_2^{\mu_2}, t_2^{\mu_2})\rangle = \langle \psi_s(\lambda_2|A_1^{\lambda_2}, t_1^{\lambda_2})\psi_s(\lambda_2|A_2^{\lambda_2}, t_2^{\lambda_2})\rangle = \tag{103}$$

$$\langle k_0 \sum_{k_{n_1}} \int dO_1 k_{n_1}^2 f(ck_{n_1}) \cos\{-k_{n_1 x} r(1-\cos\delta) - k_{n_1 y} r \sin\delta - ck_{n_1} t_1 - \theta(\vec{k}_{n_1})\} \times$$

$$k_0 \sum_{k_{n_2}} \int dO_2 k_{n_2}^2 f(ck_{n_2}) \cos\{-ck_{n_2} t_2 - \theta(\vec{k}_{n_2})\}\rangle. \tag{104}$$

On the right side of (103) the coordinates $A_1^{\lambda_2}$, $A_2^{\lambda_2}$ and the times $t_1^{\lambda_2}$, $t_2^{\lambda_2}$ of both points and the wave functions are again considered in the same reference frame λ_2. In the (104) the explicit expressions of these wave functions in the λ_2 reference frame are given.

In spherical coordinates the relationship for the massless scalar field

$$\langle \cos\theta(\vec{k}_{n_1})\cos\theta(\vec{k}_{n_2})\rangle = \langle \sin\theta(\vec{k}_{n_1})\sin\theta(\vec{k}_{n_2})\rangle = \frac{1}{2}\delta^3(\vec{k}_{n_1} - \vec{k}_{n_2}) \tag{105}$$

with a discrete spectrum becomes (compare with (45)):

$$\langle \cos[\theta(k_0 n_1 \bar{k}_1)]\cos[\theta(k_0 n_2 \bar{k}_2)]\rangle = \langle \sin[\theta(k_0 n_1 \bar{k}_1)]\sin[\theta(k_0 n_2 \bar{k}_2)]\rangle = \frac{1}{k_0 (n_1 k_0)^2}\delta_{n_1 n_2}\delta(\bar{k}_1 - \bar{k}_2). \tag{106}$$

Using this expression the correlation function can be written in the form:

$$\begin{aligned}\langle \psi_s(\mu_1|A_1^{\mu_1}, t_1^{\mu_1})\psi_s(\mu_2|A_2^{\mu_2}, t_2^{\mu_2})\rangle = k_0^2 \sum_n \int dO (nk_0)^4 f^2(cnk_0)\frac{1}{k_0(n_1 k_0)^2} \times \\ \cos\{nk_0[r\bar{k}_x(1-\cos\theta) + r\bar{k}_y \sin\delta - c(t_2 - t_1)]\}.\end{aligned} \tag{107}$$

The Lorentz-invariant spectral function $f_0(ckn_0)$ [1], (16), is

$$f^2(cnk_0) = \frac{\hbar c}{2\pi^2 n k_0}. \tag{108}$$

Using (22)we can rotate $(\bar{k}_x, \bar{k}_y, \bar{k}_z)$ to $(\bar{k}'_x, \bar{k}'_y, \bar{k}'_z)$. Then $\bar{k}_x(1-\cos\delta) + \bar{k}_y \sin\delta = 2\sin\frac{\delta}{2}\bar{k}'_y$ and we arrive to the expression:

$$\langle \psi_s(\mu_1|A_1^{\mu_1}, t_1^{\mu_1})\psi_s(\mu_2|A_2^{\mu_2}, t_2^{\mu_2})\rangle = \frac{k_0^2 \hbar c}{2\pi^2}\int do \sum_{n=0}^{\infty} n\cos nF, \tag{109}$$

where $do = d\theta d\phi \sin\theta$, F is defined in (47) and depends on both θ and ϕ.

5.2 Abel-Plana formula and the temperature of the massless scalar field observed by a rotating detector.

Abel-Plana summation formula, we have already discussed above, in this case is

$$\sum_{n=0}^{\infty} n\cos nF = \int_0^\infty dt\ t\cos tF - \int_0^\infty dt \frac{2t\cosh tF}{e^{2\pi t} - 1} \tag{110}$$

or

$$\Omega^2 \sum_{n=0}^{\infty} n\cos nF = \int_0^\infty d\omega\ \omega\cos\omega\tilde{F} - \int_0^\infty d\omega \frac{2\omega\cosh\omega\tilde{F}}{e^{\frac{\hbar\omega}{kT_{rot}}} - 1}, \tag{111}$$

where T_{rot} is defined in (54).

This expression is similar to the expression [[1]], (27) for the correlation function of the detector at rest in Planck's spectrum:

$$\int_0^\infty d\omega\omega\coth\frac{\hbar\omega}{2kT}\cos\omega t = \int_0^\infty d\omega\cos\omega t + \int_0^\infty d\omega \frac{2\omega\cos\omega t}{e^{\frac{\hbar\omega}{kT}} - 1}. \tag{112}$$

The likeness between them becomes especially close when $t = 0$ and $\tilde{F} = 0$. The appearance of the Planck's factor $(e^{\frac{\hbar\omega}{kT}} - 1)^{-1}$ in (111)points out that the rotating detector in the massless scalar zero-point field observes the same radiation spectrum as an inertial observer placed in a thermostat filled up with the radiation at the temperature $T = T_{rot}$.

APPENDIX

A λ_τ and μ_τ reference frames, Lorentz transformations, and initial condition.

We have already mentioned in the Introduction that global RF's λ_τ and μ_τ, by definition, are connected by a Lorentz transformation and agree at any proper time τ, measured by the detector clock. This initial condition is different from one used in a usual Lorentz transformation when two inertial systems agree at the time $t = t' = 0$ [26]. Let us consider the connection between these RF's in detail.

We expect that any event $(\vec{x}^\lambda, t^\lambda)$ at λ_τ RF is connected with an event $(\vec{x}^\mu, t^\mu)$ at μ_τ RF as [17], II.25′(in our notations):

$$\vec{x}^\lambda = \vec{x}^\mu + \vec{v}^\mu \frac{\vec{x}^\mu \vec{v}^\mu}{v^2}(\gamma - 1) - \vec{v}^\mu t^\mu \gamma,$$
$$t^\lambda = \gamma t^\mu - \gamma \frac{\vec{v}^\mu \vec{x}^\mu}{c^2}, \tag{113}$$

and, because the detector velocity vector

$$(v_1^\mu, v_2^\mu, v_3^\mu) = \vec{v}^\mu = -\vec{v}^\lambda = (0, -v, 0), \tag{114}$$

(note that $v_2^\mu = -v$) the equations have form:

$$x_1^\lambda = x_1^\mu, \quad x_2^\lambda = (x_2^\mu + v t^\mu)\gamma,$$
$$x_3^\lambda = x_3^\mu, \quad t^\lambda = (t^\mu + \tfrac{v}{c^2} x_2^\mu)\gamma \tag{115}$$

Under these transformations,

$x_2^\mu = 0$, $t^\mu = \tau$

transform to

$x_2^\lambda = v\tau\gamma$, $t^\lambda = \tau\gamma$

at any detector proper time τ.

The x_2^λ is not zero, and μ_τ and λ_τ do not agree, against our expectations. This could mean that our assumption that RF's, μ_τ and λ_τ agree at the proper time $\tau$is wrong or the form of the Lorentz transformation we use here is not correct.

We will now show that it is the Lorentz transformation that should be slightly modified following the new initial condition. Indeed, the equations (113) are derived with the assumption [19] that two inertial reference frames agree that is

$x_2^\lambda = x_2^\mu = 0$

at

$t^\lambda = t^\mu = 0.$

In our problem, this initial condition is true, and

$t^\lambda = t^\mu = 0,$

for only one pair of the RF's, λ_τ and μ_τ, when $\tau = 0$. It is false when $\tau \neq 0$.

It is easy to see that the modified transformation

$$x_2^\lambda = (x_2^\mu + vt^\mu)\gamma + a_\tau \qquad (116)$$
$$t^\lambda = (t^\mu + \frac{vx_2^\mu}{c^2})\gamma$$

transform

$$x_2^\mu = 0, t^\mu = \tau \qquad (117)$$

to

$$x_2^\lambda = 0, t^\lambda = \tau\gamma \qquad (118)$$

if a constant a_τ, depending on the parameter τ, is set to $-v\gamma\tau$.

This modified Lorentz transformation leaves the intervals invariant (for simplicity we use here 2-dimensional interval)

$$(x_2^\lambda)^2 - c^2(t^\lambda - \tau\gamma)^2 = (x_2^\mu)^2 - c^2(t^\mu - \tau)^2 = 0. \qquad (119)$$

It differs from the usual Lorentz transformation with the initial condition. For example the [[17]] successive Lorentz transformation has initial condition at $\tau = \tau' = 0$. So we have proved that our

assumption in the Introduction, that the rotating detector can be at the origin of both RF's, μ and λ, at any time τ, are true, and its coordinates in both RFs are:

$$A^{\mu} = (x_1^{\mu}, x_1^{\mu}, x_3^{\mu}) = 0, \quad A^{\lambda} = (x_1^{\lambda}, x_2^{\lambda}, x_3^{\lambda}) = 0 \tag{120}$$

There is a slight difference between our definition of RF and the one used in [12]. In [12], the λ and μ RF's are defined and Lorentz transformations are used in terms of orthogonal tetrades locally, at a point of the world line of a rotating detector moving in the non-Minkovskian space-time, with the metrics $g_{\mu\nu}$. They are not applied to spatial coordinates and time.

B Hyperbolic motion and Lorentz transformation.

The concept of an inertial reference frame I_τ in [1], [2] is a central part of the calculation of the correlation function in the case of an uniformly accelerating point detector. The inertial frame I_τ is defined by the condition that the point detector is instantaneously at rest in I_τ at the proper time measured by its clock. Also at time $t_\tau = \tau$ the detector position is at the origin of I_τ that is $x_\tau = 0$. Then it is assumed that a Lorentz transformation exists, from I_τ to the laboratory coordinate system, which is supposed to transform these two coordinates to $X_\star(\tau)$ and $t_\star(\tau)$, defined in (8) and (9) equations of [2].

$$\begin{aligned} X_\star(\tau) &= \frac{c^2}{a}[\cosh(\frac{a\tau}{c}) - 1] \\ t_\star &= \frac{c}{a}\sinh(\frac{a\tau}{c}) \end{aligned} \tag{121}$$

Correctness of this assumption has never been proved explicitly but nevertheless it was used directly in calculations in (17)and (53) of [1]. The coordinates $X_\star(\tau)$ and $t_\star(\tau)$ describe the motion of the detector in the laboratory coordinate system and, as we will show here, have nothing to do with the coordinates $x_{\tau\star}$ and $t_{\tau\star}$, which can be obtained as a result of a Lorentz transformation applied to $t_\tau = \tau$ and $x_\tau = 0$ in I_τ. Indeed, at time τ, velocity v_τ and γ_τ are (3 and 4 in [1]):

$$\begin{aligned} v_\tau &= c\tanh(\frac{a\tau}{c}), \\ \gamma_\tau &= (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} = \cosh(\frac{a\tau}{c}), \end{aligned} \tag{122}$$

and

$$x_{\star\tau} = (x_\tau + v_\tau t_\tau)\gamma_\tau$$

$$t_{\star\tau} = (t_\tau + \frac{v_\tau x_\tau}{c^2})\gamma_\tau. \tag{123}$$

If $x_\tau = 0$ and $t_\tau = \tau$ then

$$\begin{aligned} x_{\star\tau} &= c\tau \sinh \frac{a\tau}{c} = v_\tau \tau \gamma_\tau, \\ t_{\star\tau} &= \tau \cosh \frac{a\tau}{c} = \tau\gamma_\tau. \end{aligned} \tag{124}$$

Obviously

$$\begin{aligned} x_{\star\tau} &\neq X_\star(\tau) \\ t_{\star\tau} &\neq t_\star(\tau) \end{aligned} \tag{125}$$

Fortunately, as we will show, this assumption does not effect the final results in the case of a uniformly accelerating detector if in addition to an infinite set of instantaneous reference frames I_τ in [1], [2] we introduce an infinite set of inertial reference frames $I_{\tau\star}$, which are at rest relative to the laboratory coordinate system. This way we have two reference systems, an inertial one consisting of $I_{\tau\star}$ reference frames and an accelerating one consisting of I_τ reference frames. The first reference system is similar to the $\{\lambda_\tau\}$ reference system and the second is similar to the $\{\mu_\tau\}$ reference system described in previous sections. The locations of the accelerating detector in two reference frames I_{τ_1} and I_{τ_2} are:

$$\begin{aligned} A_1^{I_{\tau_1}} &= (0,0,0), t_1^{I_{\tau_1}} = \tau_1, \\ A_2^{I_{\tau_2}} &= (0,0,0), t_2^{I_{\tau_2}} = \tau_2. \end{aligned} \tag{126}$$

After applying the Lorentz transformation with the initial condition (116) they are transformed to their locations in the references frames $I_{\tau_1\star}$ and $I_{\tau_2\star}$:

$$\begin{aligned} A_1^{I_{\tau_1\star}} &= (0,0,0), t_1^{I_{\tau_1\star}} = \gamma_{\tau_1}\tau_1, \\ A_2^{I_{\tau_2\star}} &= (0,0,0), t_2^{I_{\tau_2\star}} = \gamma_{\tau_1}\tau_2. \end{aligned} \tag{127}$$

The references frames $I_{\tau_1\star}$ and $I_{\tau_2\star}$ have the same axis directions but are shifted against each other by the distance

$$X_\star(\tau_2) - X_\star(\tau_1) = \frac{c^2}{a}(\cosh(\frac{a\tau_2}{c}) - \cosh(\frac{a\tau_1}{c})) \tag{128}$$

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